

Quantum Measurement and Initial Conditions

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It is shown that for any given measurement settings, only some of all possible initial conditions of the observed system are compatible with those of the measurement apparatus.

This remains true in the standard formulations of quantum mechanics, both when we consider that the measurement process takes place unitarily, and when we assume non-unitary collapse. It also remains true for hidden variable theories, both deterministic and stochastic. For deterministic (including unitary) theories the condition of compatibility between the initial conditions of the observed system with those of the measurement apparatus applies indefinitely back in time. Indeterministic processes are less strict, and the condition is required to apply only starting with the most recent reset of the initial conditions. But the most recent collapse can be pushed back in time indefinitely, as shown by the delayed choice experiments.

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I. INTRODUCTION

A. Initial conditions in quantum mechanics

Quantum mechanics is usually described as consisting in two processes (von Neumann, [1]). The first one is the *unitary evolution*, or the \mathcal{U} process

$$|\psi(t)\rangle = U(t, t_0)|\psi_0\rangle, \quad (1)$$

obtained by solving the Schrödinger equation with initial condition $|\psi(t_0)\rangle = |\psi_0\rangle \in \mathcal{H}$, where \mathcal{H} is a *Hilbert space* (fig. 1).

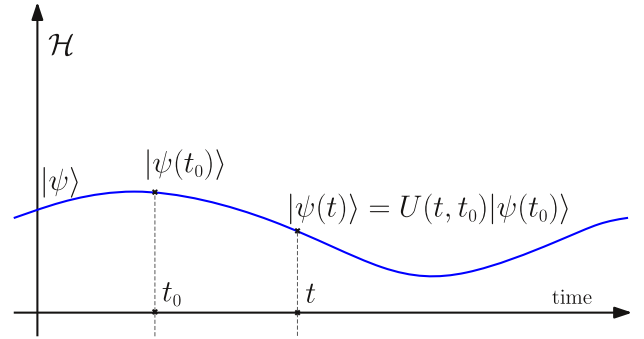


FIG. 1. The unitary evolution, or the \mathcal{U} process.

A classical system is determined by a set of partial differential equations, and initial conditions. Initial conditions are determined by an experiment performed at the time t_0 (within an error inherent to measurements). In classical mechanics, the observation process can find the system in any allowed state.

In quantum mechanics, the system can only be found in one of the eigenstates of the observable (what the measurement apparatus records is the eigenvalue). The measurement of the quantum state of the system is considered to trigger the second process, the *wavefunction collapse*, or the *state vector reduction* process \mathcal{R} (fig. 2).

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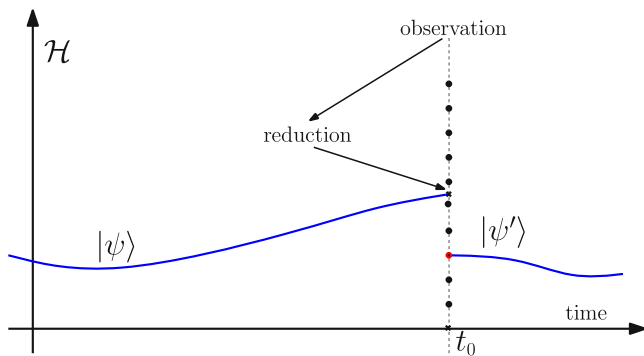


FIG. 2. The wavefunction collapse, or the \mathcal{R} process.

This consists in projecting the state of the quantum system on an eigenstate of the measured observable, resetting by this its initial conditions.

B. The density matrix formalism

More generally, we can consider instead of the state vector $|\psi\rangle \in \mathcal{H}$, a *density operator* (or matrix) ρ , which is Hermitian on \mathcal{H} . In at least one orthonormal basis $(|\psi_i\rangle)_i$, the density matrix ρ has the diagonal form

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|, \quad (2)$$

where $p_i \geq 0$, $\sum_i p_i = 1$. The density matrix can be interpreted as a *statistical ensemble* (“improper mixture”, by the terminology of d’Espagnat), where p_i is the probability that the system is in the state $|\psi_i\rangle$. It can also be understood as a *reduced density matrix* of a pure state from a higher Hilbert state (“proper mixture”), representing all the information contained in a system which is entangled with another system which is ignored.

The \mathcal{U} process for a density matrix ρ is described with the help of the time evolution operator, by

$$\rho(t) = U(t, t_0)\rho(t_0)U(t, t_0)^{-1}. \quad (3)$$

To apply the \mathcal{R} process, the density matrix should be *decohered*, that is, it should be diagonal in an eigenbasis of the observable. Then, it is interpreted as a statistical ensemble, and the probability to find the system in the state $|\psi_i\rangle$ is given by p_i .

C. The property of restricted initial conditions

Let \mathcal{S} be the set of all possible states of the observed system. For example, the states can be the rays in a Hilbert space \mathcal{H} , or density matrices, or, for hidden variable theories, they may contain hidden variables. Let $\mathcal{S}_\mathcal{O}$ be the set of all possible states of the observed system before the measurement of the observable \mathcal{O} , for which

the measurement gives a definite outcome. Such states are said to be *compatible* with the measurement.

Property 1 (of restricted initial conditions). There are states in \mathcal{S} which are not in $\mathcal{S}_\mathcal{O}$.

In other words, not all possible initial states of the observed system are compatible with a given observable. In this paper we will show that quantum mechanics, and any theory which aims to reproduce its predictions, has the property of restricted initial conditions.

D. Brief outline

The \mathcal{R} process, as it is usually presented, violates the \mathcal{U} process. There is an increasingly popular opinion that the unitary evolution of the observed system, plus the measurement apparatus, plus the *environment*, can explain the apparently non-unitary \mathcal{R} process. We will discuss this in section §II. We will see that, if we consider that quantum measurement doesn’t violate unitary evolution, then the theory has the Property 1, of restricted initial conditions.

In section §III A we will check if a discontinuous or non-unitary collapse can avoid this property. The initial conditions behave as in the unitary case, but only since the most recent wavefunction collapse. But the experimental set-up can be made so that the choice of the observable is delayed indefinitely, suggesting that the most recent collapse could happen long time before the measurement. Also, weak measurements of the observed system seem to anticipate the strong measurements. Hence, Property 1 is still true.

It is also true for hidden variable theories, both deterministic and stochastic, as we know from Bell’s theorem and other theorems of this type (section §III B).

Section §IV analyses possible physical interpretations of the property of restricted initial conditions.

II. IF THE COLLAPSE IS ASSUMED TO BE UNITARY

A. Is unitary evolution violated during measurement?

Unitary evolution seems to be ubiquitous. The exception, and the reason for the introduction of the \mathcal{R} process, is that when the system is in a certain state, a subsequent measurement may project it to a different state. But one does not exclude the possibility that, when we consider in addition to the observed system, the environment (including the measurement apparatus and the observer) and the interactions between these systems, the evolution turns out to be unitary.

The viewpoint that unitary evolution is enough, and can account for the \mathcal{R} process too, gained more and more supporters lately, due to the development of the Many

Worlds Interpretation (MWI) [2–7], the *consistent histories* interpretation [8–14], and especially of the *decoherence program* [15–24].

A pole took place at a conference on quantum computation, at the Isaac Newton Institute in Cambridge, in July 1999. The question “*Do you believe that all isolated systems obey the Schrödinger equation (evolve unitarily)?*” received 59 answers of “yes”, 6 of “no”, and the remaining 31 physicists were undecided. Tegmark and Wheeler commented about this [25]

although these [quantum textbooks] infallibly list explicit non-unitary collapse as a fundamental postulate in one of the early chapters, the poll indicates that many physicists – at least in the burgeoning field of quantum computation – no longer take this seriously.

The decoherence approach is based on the idea that the interactions with the environment cause the density matrix of the observed system to become diagonal, in a preferred basis. After diagonalization, which is viewed as a *pre-measurement*, the density matrix can be interpreted as a statistical ensemble. Presumably, once two branches decohered, they no longer interact with one another, this leading to an “effective collapse”, which would replace the \mathcal{R} process.

Other proposal that for quantum mechanics the \mathcal{U} process is enough, even for the apparent \mathcal{R} process, was made by the author in [26, 27], and another one by ‘t Hooft in [28].

Regarding the universality of the \mathcal{U} process, the opinions are divided. More details, and deep analyses of these opinions, are listed and discussed in [22, 24, 29]. Therefore, it is justified to consider the hypothesis:

Hypothesis 1. The \mathcal{R} process is reducible to the \mathcal{U} process.

We will show that from this hypothesis follows that only a small part of the possible initial states of the observed system can evolve in eigenstates of the observable.

B. Unitary collapse and initial conditions

1. From outcomes, to initial conditions

If we take the eigenstate obtained by the \mathcal{R} process back in time, by reversing the unitary evolution, we can interpret the same result as if the unitary evolution was never violated, and the projection actually happened at the beginning, in the very initial conditions. This works both for a state vector, and for a decohered density matrix which we interpret as a statistical ensemble.

The price to be paid is that relying solely on the \mathcal{U} process requires very special initial conditions. The initial conditions had to be from the very *beginning* so special, so that *later*, when the measurement is performed, an eigenstate of the observable is obtained (fig. 3).

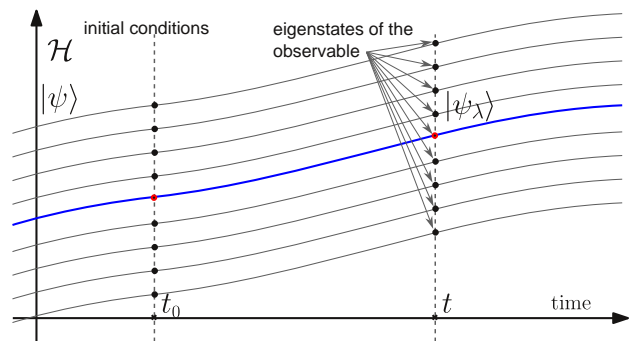


FIG. 3. Unitary evolution requires the initial state to be special, so that the measurement finds the system in an eigenstate of the observable.

This can be seen, intuitively, if we decide to observe a different property of the system, and change the settings in the last moment (Wheeler’s *delayed choice experiment* [30–34]).

Next we will give a rigorous proof that if the \mathcal{R} process is just an effect of the \mathcal{U} process, then the initial conditions have to be very special, *i.e.* Property 1 holds.

2. Unitary evolution, measurement, and initial conditions

Can measurements make a quantum system, just by unitary evolution, to become an eigenvalue of the observable, for any initial state of the observed system? More precisely, let $|\psi\rangle \in \mathcal{H}_\psi$ be a quantum system, and \mathcal{O} an observable corresponding to the system. Assume that the observable is measured by a system $|\mu\rangle \in \mathcal{H}_\mu$, which is considered to be a quantum system. We can consider the measurement apparatus $|\mu\rangle$ as containing the environment, in the sense of the decoherence program. It is often claimed that this ingredient can help the observed system to become an eigenstate of the observable. We are interested if it is possible that the following conditions are simultaneously satisfied:

1. Initially, the observed system and the measurement apparatus are considered to be separated. The measurement apparatus should have no prior “knowledge” about the observed system, or to be entangled with it before the measurement.
2. The measurement performed by $|\mu\rangle$ makes the observed system, for any possible initial state $|\psi\rangle \in \mathcal{H}_\psi$, by disturbing it if necessary, to be an eigenstate of the observable \mathcal{O} .
3. For any eigenvalue λ of the observable, there is an initial value of the observed system, so that the outcome of the measurement is λ .
4. This is achieved by unitary evolution only.

The following simple theorem shows that in general it is not possible to satisfy all these conditions.

Theorem 1. Let \mathcal{H}_μ and \mathcal{H}_ψ be two separable Hilbert spaces. Let $U : \mathcal{H}_\mu \otimes \mathcal{H}_\psi \rightarrow \mathcal{H}_\mu \otimes \mathcal{H}_\psi$ be a unitary operator, \mathcal{O} a Hermitian operator on \mathcal{H}_ψ , and let $|\mu\rangle \in \mathcal{H}_\mu$ be fixed. Suppose that for every $|\psi\rangle \in \mathcal{H}_\psi$, $U(|\mu\rangle|\psi\rangle)$ has the form

$$U(|\mu\rangle|\psi\rangle) = |\mu'\rangle|\psi'\rangle, \quad (4)$$

where $|\psi'\rangle$ is an eigenvector of the Hermitian operator \mathcal{O} . Then, all possible state vectors $|\psi'\rangle$ correspond to the same eigenvalue of \mathcal{O} .

Proof. Let $|\psi'_1\rangle$ and $|\psi'_2\rangle$ be two orthogonal eigenvectors in \mathcal{H}_ψ , so that

$$U(|\mu\rangle|\psi_1\rangle) = |\mu'_1\rangle|\psi'_1\rangle \quad (5)$$

and

$$U(|\mu\rangle|\psi_2\rangle) = |\mu'_2\rangle|\psi'_2\rangle \quad (6)$$

for some vectors $|\psi_1\rangle, |\psi_2\rangle \in \mathcal{H}_\psi$. Because $|\psi'_1\rangle$ and $|\psi'_2\rangle$ are orthogonal, $|\mu'_1\rangle|\psi'_1\rangle$ and $|\mu'_2\rangle|\psi'_2\rangle$ are orthogonal too. Since U is unitary, it follows that $U^{-1}(|\mu'_1\rangle|\psi'_1\rangle)$ and $U^{-1}(|\mu'_2\rangle|\psi'_2\rangle)$ are orthogonal too, and since they are just $|\mu\rangle|\psi_1\rangle$ and $|\mu\rangle|\psi_2\rangle$, it follows that $|\psi_1\rangle$ and $|\psi_2\rangle$ are orthogonal as well. For any two complex numbers α_1 and α_2 ,

$$U(|\mu\rangle(\alpha_1|\psi_1\rangle + \alpha_2|\psi_2\rangle)) = \alpha_1|\mu'_1\rangle|\psi'_1\rangle + \alpha_2|\mu'_2\rangle|\psi'_2\rangle. \quad (7)$$

But the hypothesis requires that there are $|\mu''\rangle \in \mathcal{H}_\mu$ and an eigenvector $|\psi''\rangle \in \mathcal{H}_\psi$ of \mathcal{O} , so that

$$\alpha_1|\mu'_1\rangle|\psi'_1\rangle + \alpha_2|\mu'_2\rangle|\psi'_2\rangle = |\mu''\rangle|\psi''\rangle. \quad (8)$$

Because $\langle\psi'_1|\psi'_2\rangle = 0$, this can only happen if $|\mu'_2\rangle = \beta|\mu'_1\rangle$ for some $\beta \in \mathbb{C}$. But then,

$$|\mu'_1\rangle(\alpha_1|\psi'_1\rangle + \alpha_2\beta|\psi'_2\rangle) = |\mu''\rangle|\psi''\rangle. \quad (9)$$

From this it follows that for any $\alpha_1, \alpha_2 \in \mathbb{C}$, the linear combination $\alpha_1|\psi'_1\rangle + \alpha_2\beta|\psi'_2\rangle$ is an eigenvector. But this can only happen if the eigenvectors $|\psi'_1\rangle$ and $|\psi'_2\rangle$ correspond to the same eigenvalue. This concludes the proof. \square

Remark 1. This means that, if the system made of the measurement apparatus (including any environment we wish) and the observed system evolves unitarily, the outcome can be an eigenstate of the observable only for particular initial conditions of the observed system. No matter what the environment is, and how we hope it affects the observed system, as long as evolution is unitary, the conclusion of the theorem can't be avoided. Property 1, of restricted initial conditions, is satisfied.

Remark 2. One may think that if we would use density matrices instead of vectors to represent the quantum systems, the gain in generality would allow us to avoid the conclusion of Theorem 1. It is easy to see that this is not the case. The reason is that a generic density matrix has the form $\rho = \sum_i \sum_j p_{ij} |\psi_i\rangle\langle\psi_j|$, where $|\psi_i\rangle \in \mathcal{H}_\psi$. Since the vectors $|\psi_i\rangle$ transform by the unitary evolution to eigenvectors of \mathcal{O} corresponding to the same eigenvalue λ , the density matrix transforms to a density matrix on the eigenspace corresponding to the same eigenvalue λ . So this generality is not enough to avoid the conclusion of the theorem.

3. Unitary measurement apparatus

Theorem 1 refers to any system $|\mu\rangle$ which may put the observed system $|\psi\rangle$ in an eigenstate of the observable. But normally a measurement apparatus, in addition, is required to leave unchanged the states which are already eigenstates of the observable.

Such a hypothetic apparatus which works unitarily was discussed for example by Zurek in [35], p. 195–196, where he proved that measurement can only distinguish orthogonal states.

Theorem 1 applies to such an apparatus, but the condition that the eigenstates of the observable are left unchanged by the measurement process is even more strict about the admissible initial conditions, as the following simple result shows.

Theorem 2. In addition to the conditions from Theorem 1, suppose that for each eigenstate $|\psi_i\rangle \in \mathcal{H}_\psi$ of the observable \mathcal{O} , there is a state vector $|\mu_i\rangle \in \mathcal{H}_\mu$, so that $U(|\mu_i\rangle|\psi_i\rangle) = |\mu_i\rangle|\psi_i\rangle$. Then, the only initial states $|\psi\rangle$ which are compatible with the measurement (*i.e.* become eigenstates of the observable) are those which already were eigenstates before the measurement.

Proof. Let $|\psi\rangle \in \mathcal{H}_\psi$ be a state vector. According to conditions in Theorem 1, $U(|\mu\rangle|\psi\rangle)$ has the form $U(|\mu\rangle|\psi\rangle) = |\mu'\rangle|\psi'\rangle$ for some eigenstate $|\psi'\rangle$ of \mathcal{O} . Let $(|\psi_i\rangle)_i$ be an eigenbasis, so that for a particular j , $|\psi_j\rangle = |\psi'\rangle$. Then, we can write $|\psi\rangle = \sum_i \alpha_i |\psi_i\rangle$, and from the hypothesis and because of linearity,

$$U(|\mu\rangle|\psi\rangle) = \sum_i \alpha_i U(|\mu\rangle|\psi_i\rangle) = \sum_i \alpha_i |\mu_i\rangle|\psi_i\rangle. \quad (10)$$

Hence

$$\sum_i \alpha_i |\mu_i\rangle|\psi_i\rangle = |\mu'\rangle|\psi_j\rangle. \quad (11)$$

It follows that

$$\sum_{i \neq j} \alpha_i |\mu_i\rangle|\psi_i\rangle + (\alpha_j |\mu_j\rangle - |\mu'\rangle)|\psi_j\rangle = 0. \quad (12)$$

From the orthonormality of the eigenbasis $(|\psi_i\rangle)_i$, it follows that the nonvanishing terms appearing in the

equation (12) are linear independent vectors. Therefore, each one of them has to be zero. This means that $\alpha_j |\mu_j\rangle = |\mu'\rangle$, and for any $i \neq j$, $\alpha_i = 0$. It follows that the only initial state vectors $|\psi\rangle$ which are compatible with the measurement are already eigenstates of \mathcal{O} . \square

C. Discussion

1. What this result tells us

In classical mechanics, the observed system can start by being in any possible state. In quantum mechanics the measurement can't find the system in any state, but only in an eigenstate of the observable. One may hope that, while the observed system was initially in any possible state, the measurement disturbed it until it became an eigenstate. Theorem 1 shows that no unitary evolution, even containing the interaction with the environment, can accomplish this for any initial state. Moreover, Theorem 2 shows that if we consider a measurement apparatus which doesn't disturb eigenstates of the observable, then these are the only possible initial states compatible with the measurement. The observed system had to be already in a special state before measurement. For each initial state of the measurement apparatus (including observer and environment) only a part of the possible initial states of the observed system can lead to eigenstates of the observable. The reciprocal is also true. If one initial state may be compatible with an observable, there are other observables with which it is not compatible. Only some measurement settings are compatible with a particular initial state of the observed system.

2. Connections with other results

Theorem 1 shows that the possible initial states of a quantum system assumed to remain unitary during the measurement depend on the observable we choose to measure, *i.e.* on the experimental setup. The proof relies only on the state vector, without any assumptions about its interpretation.

Theorem 1 is in line with other similar results, like Bell's theorem [36], and Kochen-Specker's [37] theorem.

It can also be related to Wheeler's *delayed choice experiment* [30–34]. The observer can delay the choice of the observable, until the photon passes by the first beam splitter of the Mach-Zehnder interferometer (fig. 4). Wheeler showed that, if the observer chooses to perform a “which-way” measurement, the photon traveled on one of the two paths A and B, while if she chooses to measure the interference, the photon traveled both ways. This suggests that the system had an initial state compatible with the observable, even if at that time the observable was not yet known.

Theorems 1 and 2 apply to the unitary evolution of the total system, including the environment. Their proofs

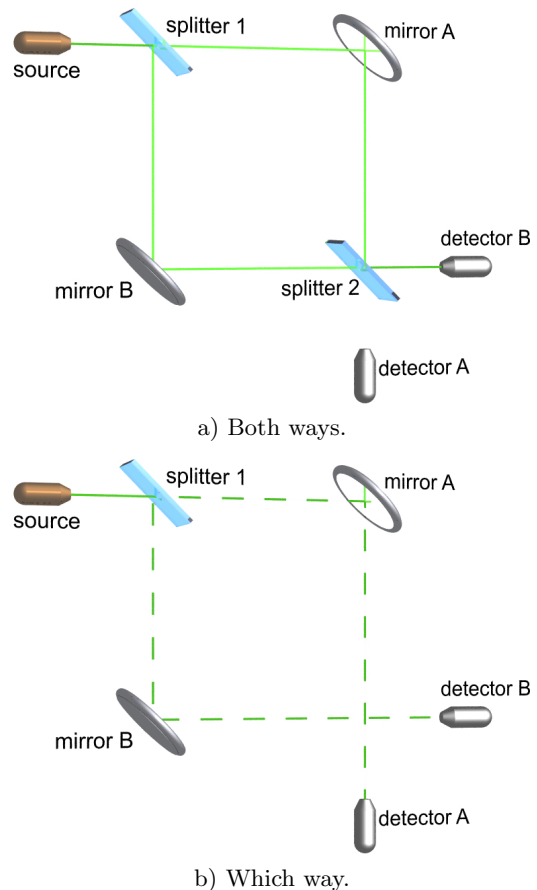


FIG. 4. Delayed choice experiment with Mach-Zehnder's interferometer.

don't rely on simultaneous measurement of entangled systems, one measurement is enough.

III. ALTERNATIVES

A. Discontinuous or non-unitary collapse

We have seen that unitary evolution can't explain the outcome of measurements without having to admit that the initial conditions of the observed system were already compatible with the observable. One may think that if we admit that the \mathcal{R} process is really discontinuous or non-unitary, the problem vanishes.

But the delayed choice experiment suggests that if there is a non-unitary collapse, the experimental set-up can be arranged so that the collapse happened before the observer decides what observable to measure.

What a non-unitary collapse does is to “reset” the states, hence the initial conditions, of the observed systems. After the collapse, the evolution is unitary, and the new initial conditions have to be compatible with the choice of the observable. Therefore, after the collapse, Property 1, of restricted initial conditions, holds. The

only advantage of the non-unitary \mathcal{R} is that we can consider that the initial conditions were free, and even could have been initially incompatible, and that the measurement resets them to make them compatible, at a more recent time. But this more recent time can be pushed back in time indefinitely.

Notable with respect to the way past seems to be influenced by future choices is the *two-state vector formalism* [38–46]. This way of describing things, by combining weak measurement with a description of quantum mechanics based on a state vector evolving in the future, and another one in the past, reveals something intimate about the nature of quantum mechanics. It proved to be very proficient in thinking about new experiments, which explore the limits we thought quantum mechanics has. For example, in relation with the idea of apparent influence of the past by the future, in [46] is revealed how the EPR experiment can be modified to show that apparently future strong measurements affect the outcomes of weak measurements performed in the past.

Apparently non-local and non-causal behaviors suggest that even the version of quantum mechanics where the collapse is admitted to be non-unitary or discontinuous, also requires the initial conditions to be very special, compatible with the initial conditions of other systems participating in the experiment. Hence, the property of restricted initial conditions cannot be avoided by admitting a non-unitary \mathcal{R} .

B. Hidden variables

Could hidden variables theories give the same predictions as quantum mechanics (which are confirmed by so many experiments), and yet be formulated in terms of fundamental entities which escape the property of restricted initial conditions?

The answer follows immediately from the well known results, due to the EPR experiment [47, 48], and to Bell’s theorem [36]. Bell’s theorem and others, like Kochen-Specker’s, show that whatever parameters we would use, at least in the case of spin and polarization, they cannot decide the outcome of the measurement, in a way which is independent on the observables. The conclusions of these theorems are usually presented in a way which emphasizes the impossibility of local hidden variable theories, and in discussions about Einstein’s criterion of reality. They say that the hidden variables have to depend on the choice of the direction along which we measure, hence on the experimental set-up. Therefore, hidden variable theories, both the deterministic ones, and those with a stochastic element (corresponding to the \mathcal{R} process), have the Property 1, of restricted initial conditions.

IV. INTERPRETATIONS

What does it mean that, for the evolution to remain unitary during measurement, the initial conditions have to be very special? Does this mean that the initial conditions “guess” the future choice of observable and the interaction with the measurement apparatus? One possible view is that the initial conditions of the observed system are not decided until the observer chooses the measurement apparatus. In other words, the initial conditions themselves are delayed. This may contradict our intuition on causality.

The second possible interpretation is that the observer is “predestined” to choose the observable, so that the outcome is an eigenvalue of that observable (see *e.g.* [28]). This explanation is called *superdeterminism*. Apparently, it denies the free will of the observer (we will not argue here if we should be concerned about the free will or not. The interested reader may consult [27]).

What would be more acceptable, to admit that the observed system is “predestined” to become an eigenstate of the observable, or that the observer is “predestined” to choose an observable which is compatible with the observed system? Both interpretations are different than what one would expect causality to be like.

Another possibility is to consider that the initial conditions of the observed system and the measurement apparatus were already entangled, prior to the measurement (in contradiction with the hypothesis of Theorem 1). For example, in the case of the experiment with the Mach-Zehnder interferometer (fig. 4), the measurement apparatus can be arranged in two ways, $|\text{observe both-ways}\rangle$ and $|\text{observe which-way}\rangle$, and the system can be found either in the $|\text{both-ways state}\rangle$, or in one of the states $|\text{which-way state}\rangle_A$ and $|\text{which-way state}\rangle_B$ (depending whether the photon went through the arm labeled A , or the arm labeled B , in fig. 4). The total state will be therefore of the form

$$\begin{aligned} |\text{total state}\rangle = & \alpha |\text{observe both-ways}\rangle |\text{both-ways state}\rangle \\ & + \frac{\beta}{\sqrt{2}} |\text{observe which-way}\rangle_A |\text{which-way state}\rangle_A \\ & + \frac{\beta}{\sqrt{2}} |\text{observe which-way}\rangle_B |\text{which-way state}\rangle_B \end{aligned} \quad (13)$$

where $|\alpha|^2$ is the probability that a both-ways measurement is performed, and $|\alpha|^2 + |\beta|^2 = 1$. We can consider this a statistical ensemble, and eventually find that only one of the states in the superposition is realized, depending on the choice of the observable. This reformulation in terms of entanglement may seem more legit, but unfortunately the decomposition in equation (13) is not unique. More important, it is not hard to see that only a subset of measure zero of $\mathcal{H}_\mu \otimes \mathcal{H}_\psi$ is allowed for the initial conditions. Hence, the problem remains the same: the initial conditions have to be very special in a way which seems to anticipate the future choices. We can’t avoid Property 1, of restricted initial conditions.

V. CONCLUSION

We have seen that, if we assume only unitary evolution, a particular initial state of the measurement apparatus is consistent only with a particular set of possible initial states of the observed systems. In other words, to allow unitary evolution during measurements, the initial conditions of quantum systems have to be constrained in a way which depends on future choices of the observable.

The initial conditions of various systems involved in a quantum experiment have to be compatible, before the experiment. This happens in the case of hidden variable

theories, as follows from the Bell and Kochen-Specker theorems, but it happens also when we consider quantum mechanics without additional variables. It happens if we consider that the evolution is always unitary and the collapse is only apparently non-unitary (section §II B 2), but also when we admit an authentic discontinuous or non-unitary collapse.

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